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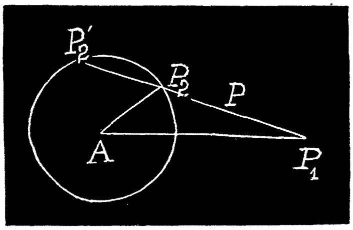
**319. Proposed by S. F. NORRIS, Baltimore City College, Baltimore, Md.**

Lines are drawn from a fixed point $P_1$, meeting a fixed circle in $P_2$. On $P_1P_2$ a point $P$ is taken so that $P_1P\times P_1P_2=k^2$. Find the locus of $P$. Solve by analytic methods, using rectangular coordinates, and putting the result in the form,

$$(x_1^2+y_1^2-r^2)[(x-x_1)^2+(y-y_1)^2]+2k^2(x_1x+y_1y-x_1^2-y_1^2)+k^4=0.$$

**Solution by J. SCHEFFER, A. M., Hagerstown, Md.**



Let $AP_1$ be the axis of abscissas, $A$ the origin of orthogonal coordinates, $x$ and $y$ the co-ordinates of $P$, $\phi$ the $\angle P_2P_1A$, and $\theta$ the $\angle AP_2P'_2$. Then $PP_1=\frac{y}{\sin\phi}$, $P_1P_2=x_1\cos\phi-r\cos\theta$. Therefore from the condition $PP_1\times P_1P_2=k^2$ we have $\frac{y}{\sin\phi}(x_2\cos\phi-r\cos\theta)=k^2$, but $\tan\phi=\frac{y}{x_1-x}$. $\therefore\sin\phi=\frac{y}{\sqrt{[y^2+(x_1-x)^2]}}$, $\cos\phi=\frac{x_1-x}{\sqrt{[y^2+(x_1-x)^2]}}$, $\sin\theta=\frac{x_1\sin\phi}{r}$. Substituting, we get $\frac{y}{\sin\phi}[x_1\cos\phi-\sqrt{(r^2-x_1^2\sin^2\phi)}[=k^2$, or, removing the radical, $x_1^2y^2-2k^2x_1\sin\phi\cos\phi\, y+k^4\sin^2\phi=r^2y^2$, and substituting now the values for $\sin\phi$ and $\cos\phi$ we obtain, after some reductions that admit of no difficulty,

$$y^2+\left[x+\frac{k^2+r^2-x_1^2}{x_1^2-r^2}x_1\right]^2=\frac{k^4r^2}{(x_1^2-r^2)^2},$$

or putting $x_1^2-r^2=a^2$=square of the tangent drawn from $P_1$,

$$y^2+\left[x+\frac{k^2-a^2}{a^2}x_1\right]^2=\frac{k^4r^2}{a^4},$$

the equation of a circle, whose center lies on $AP_1$, at a distance $=\frac{a^2-k^2}{a^2}x_1$ from $A$, and whose radius$=\frac{k^2r}{a^2}$.

Also solved by G. B. M. Zerr.

## CALCULUS.

**247. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.**

Integrate, $x\frac{\partial^2y}{\partial x^2}+2\frac{\partial y}{\partial x}-xy=0.$

**Solution by GEORGE W. HARTWELL, Columbia University, New York.**

Dividing by $x$, $\dfrac{\partial^2 y}{\partial x^2}+\dfrac{2}{x}\dfrac{\partial y}{\partial x}-y=0$...(1).

Remove the term containing $\dfrac{\partial y}{\partial x}$ by substituting $y=\dfrac{v}{x}$. Then (1) becomes $\dfrac{\partial^2 v}{\partial x^2}-v=0$...(2).

The solution of this equation is $v=c_1e^x+c_2e^{-x}$. $\therefore xy=(c_1e^x+c_2e^{-x})$.

**Solved in a similar manner by G. B. M. Zerr.**

**248. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.**

Evaluate $\int_0^{\frac{1}{2}\pi}\sin nx\cot x dx$, where $n$ is a positive integer.

**Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.**

$$A=\int_0^{\frac{1}{2}\pi}\sin nx\ \cot x\ dx=\int_0^{\frac{1}{2}\pi}\frac{\sin(n-1)xdx}{\sin x}+\int_0^{\frac{1}{2}\pi}\cos nx\ dx$$

$$=2\int_0^{\frac{1}{2}\pi}[\cos x+\cos 3x+\cos 5x+\ldots+\cos(n-3)x]dx+\int_0^{\frac{1}{2}\pi}\cos nx\ dx$$

$$=2\left[\sin x+\tfrac{1}{3}\sin 3x+\tfrac{1}{5}\sin 5x+\ldots+\frac{1}{n-3}\sin\,(n-3)\,x\right]_0^{\frac{1}{2}\pi}+\frac{1}{n}\cos\frac{\pi n}{2}$$

$$=2\left[1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{9}-\ldots\pm\frac{1}{n-3}\right]\pm\frac{1}{n};\ \text{for } n \text{ even}=\tfrac{1}{2}\pi \text{ if } n=\infty.$$

$$A=\int_0^{\frac{1}{2}\pi}[1+2\cos 2x+2\cos 4x+2\cos 6x+\ldots+2\cos(n-3)x]dx$$

$$+\int_0^{\frac{1}{2}\pi}\cos nx\ dx=\left[x+\sin 2x+\tfrac{1}{2}\sin 4x+\tfrac{1}{3}\sin 6x+\ldots+\frac{2}{n-3}\sin(n-3)x\right]_0^{\frac{1}{2}\pi}$$

$$+\frac{1}{n}\cos\frac{\pi n}{2}=\tfrac{1}{2}\pi,\ \text{for } n \text{ odd}.$$

**249. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.**

Ike, running with constant velocity $v$, is trying to catch Jim, running with constant velocity $V$, $(V>v)$, by keeping Jim dead ahead of him. Find their paths.